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Effective CP violation in the Standard Model

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ABSTRACT: We study the strength of effective CP violation originating from the CKM matrix in the effective action obtained by integrating out the fermions in the Standard Model. Using results obtained by Salcedo for the effective action in a general chiral gauge model, we find that there are no CKM CP-violating terms to fourth order in a gauge-covariant derivative expansion that is non-perturbative in the Higgs field. The details of the calculation suggest that, at zero temperature, the strength of CP violation is approximately independent of the overall scale of the Yukawa couplings. Thus, order of magnitude estimates based on Jarlskog's invariant could be too small by a factor of about 10^{17}.

KEYWORDS: Baryogenesis, CP-violation, Effective actions.

Contents

1. Introduction

The weak, electromagnetic and strong interactions described by the Standard Model (SM) have played a role in the shaping of the universe as we know it [1]. It is natural to assume that the same is true for finer details, such as the CP violation embodied in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2, 3]. Particularly striking is the fact that this CP violation, which is compatible with experiment [4], can occur only with three or more families, and three families are being observed. Yet, it is often stated that the CP-violation caused by the CKM matrix is too weak to be able to play a significant role in the generation of the baryon asymmetry in the universe.

A scenario that has received considerable attention over the years is electroweak baryogenesis [5, 6, 7], in which the asymmetry is supposed to be generated during the electroweak transition. One way to approach the problem of dealing with complicated non-perturbative dynamics is to concentrate on the bosonic variables by 'integrating out the fermions'. CP violation then enters the description effectively through higher-dimensional terms in an effective lagrangian. The simplest of these has been assumed to have the form [8, 9]

$$\frac{3\delta_{\mathrm{CP}}}{16\pi^2 M^2}\,\varphi^\dagger\varphi\;\mathrm{tr}\,(A^{\mu\nu}\tilde{A}_{\mu\nu}), \tag{1.1}$$

where $A_{\mu\nu}$ is the SU(2) field strength tensor, $\tilde{A}_{\mu\nu}$ its dual, M is a mass depending on the scale of the problem and δ_{CP} is a dimensionless constant characterizing the strength of the

induced CP violation. In case of the finite-temperature electroweak transition, a natural choice for M is the temperature T, and the usual estimate for $\delta_{\rm CP}$ is given by [10, 8, 9]

$$\delta_{\rm CP} = J\,(m_u^2-m_c^2)(m_c^2-m_t^2)(m_t^2-m_u^2)(m_d^2-m_s^2)(m_s^2-m_b^2)(m_b^2-m_d^2)/T^{12} \approx 10^{-19}, \quad (1.2)$$

where m_u, …,m_b are the quark masses[1], we used $T = 100$ GeV, and [4]

$$J = |{\rm Im}(V_{fg}V_{hi}V_{fi}^{*}V_{hg}^{*})| = (2.88 \pm 0.33) \times 10^{-5} \quad (1.3)$$

is the simplest rephasing-invariant combination of the CKM matrix V [11, 12]. Since the above estimate is many orders of magnitude smaller that the baryon asymmetry $n_B/n_\gamma \simeq 6 \times 10^{-10}$, the usual conclusion is that CKM CP-violation cannot have been instrumental in early universe baryogenesis.

Recently, new scenarios for electroweak baryogenesis have been put forward in which the electroweak transition is supposed to have been a tachyonic one at the end of inflation, in which the effective squared Higgs mass parameter turned negative in the early universe, *not* due to a change in temperature but because of the coupling to a changing inflaton field [13, 14, 15, 16]. At the end of electroweak-scale inflation [17, 18] the temperature is supposed to be zero, whereas the dynamics in tachyonic transitions is dominated by the low-momentum modes of the fields [19, 20]. This suggests reconsidering the above order of magnitude estimate for $\delta_{\rm CP}$ in an environment at zero temperature. Then the quark masses in (1.2) are to be replaced by the Yukawa couplings λ_u, …, λ_t ($\lambda_u = \sqrt{2}\,m_u/v$, etc., $v = 246$ GeV), giving

$$\delta_{\rm CP} = J\,(\lambda_u^2-\lambda_c^2)(\lambda_c^2-\lambda_t^2)(\lambda_t^2-\lambda_u^2)(\lambda_d^2-\lambda_s^2)(\lambda_s^2-\lambda_b^2)(\lambda_b^2-\lambda_d^2) \approx 10^{-22}, \quad (1.4)$$

even smaller than (1.2). But what to use for M? A natural choice is the (renormalized) expectation value of the Higgs field $\langle\varphi^\dagger\varphi\rangle$. In a low-temperature tachyonic electroweak transition this increases from zero to close to its vacuum expectation value $v^2/2$, suggesting a boost of the resulting CP violation when $\langle\varphi^\dagger\varphi\rangle$ is small.

However, even with the Higgs field settled in its v.e.v., the measured CP violating effects in accelerator experiments are at a much higher level than 10^{-23} [4]. This suggests that the above order of magnitude estimates of $\delta_{\rm CP}$ are misleading, at least at zero temperature (see also [21, 22, 23, 24]).

In this article we investigate CP-violation induced by the CKM matrix using results for the effective bosonic action in a general chiral gauge theory obtained by Salcedo [25, 26]. He presented remarkably explicit results to fourth order in a gauge-covariant derivative expansion, with coefficient-functions that are non-perturbative in the Higgs field. Specializing these results to the case of the SM we found that they do not contain CKM CP-violation (unfortunately). However, the general form of the results suggests strongly that the magnitude of the CP violation to be expected in higher order is primarily set by the CKM-invariant J in (1.3) and not by the tiny product of Yukawa couplings in (1.4).

In section 2 we review the results of Salcedo that are relevant for our purpose and apply these to the SM case in section 3, in so far at they are relevant to CKM CP violation.

[1] We use $m_u = 0.0025$, $m_d = 0.0045$, $m_s = 0.09$, $m_c = 1.26$, $m_b = 4.26$, $m_t = 175$ GeV.

Considerations on the magnitude of CKM CP-violation are in section 4. In section 5 we show how the CP-violating QCD θ-term may be uncovered from the effective action and our conclusions in section 6. In the appendix we give details some details of the functions calculated by Salcedo.

2. Salcedo's results

Salcedo calculated the fermion contribution to the euclidean effective action for the Bose fields in a derivative expansion up to fourth order in the gauge-covariant derivatives. The effective action, W, corresponds to a model with n Dirac fields and is formally given by

$$W = -\mathrm{Tr}\,(\ln D), \tag{2.1}$$

where D is a Dirac operator of the form

$$D = D^{\mathrm{R}}_\mu \gamma_\mu P_{\mathrm{R}} + D^{\mathrm{L}}_\mu \gamma_\mu P_{\mathrm{L}} + m_{\mathrm{LR}} P_{\mathrm{R}} + m_{\mathrm{RL}} P_{\mathrm{L}}, \tag{2.2}$$

$$P_{\mathrm{R}} = \frac{1}{2}(1+\gamma_5), \quad P_{\mathrm{L}} = \frac{1}{2}(1-\gamma_5), \tag{2.3}$$

with[2] $\gamma_\mu = \gamma_\mu^\dagger$ and $\gamma_5 = \gamma_5^\dagger = -\gamma_1\gamma_2\gamma_3\gamma_4$. The covariant derivatives $D^{\mathrm{L,R}}_\mu = \partial_\mu + v^{\mathrm{L,R}}_\mu$ depend on chiral $U(n)\times U(n)$ gauge fields $v^{\mathrm{L,R}}_\mu = -(v^{\mathrm{L,R}}_\mu)^\dagger$, and m_{LR} and $m_{\mathrm{RL}} = (m_{\mathrm{LR}})^\dagger$ are $n \times n$ matrix scalar fields transforming under gauge transformations as

$$m_{\mathrm{LR}} \to \Omega_{\mathrm{L}} m_{\mathrm{LR}} \Omega_{\mathrm{R}}^\dagger, \quad m_{\mathrm{RL}} \to \Omega_{\mathrm{R}} m_{\mathrm{RL}} \Omega_{\mathrm{L}}^\dagger. \tag{2.4}$$

We will also encounter the field strengths and covariant derivatives

$$F^{\mathrm{L,R}}_{\mu\nu} = [D^{\mathrm{L,R}}_\mu, D^{\mathrm{L,R}}_\nu], \quad \hat{D}_\mu m_{\mathrm{LR}} = \partial_\mu m_{\mathrm{LR}} + v^{\mathrm{L}}_\mu m_{\mathrm{LR}} - m_{\mathrm{LR}} v^{\mathrm{R}}_\mu, \tag{2.5}$$

and $\hat{D}_\mu m_{\mathrm{RL}} = (\hat{D}_\mu m_{\mathrm{LR}})^\dagger$.

The fermion fields form an anomalous representation of the gauge group and consequently W contains a chiral anomaly. It is not gauge invariant under the full $U(n)\times U(n)$ group but it will be so when restricted to the gauge group of the Standard Model.

The effective action can be split into terms that are even and odd under the pseudo-parity transformation $v^{\mathrm{L}} \leftrightarrow v^{\mathrm{R}}$, $m_{\mathrm{LR}} \leftrightarrow m_{\mathrm{RL}}$,

$$W = W^+ + W^-. \tag{2.6}$$

The 'normal parity' component W^+ is formally identical to the effective action of a vector-like model (see e.g. [25]). We are interested in the 'abnormal parity' component W^-, since it is odd in the number of γ_5 matrices and will contain the leading CP-violating terms. It contains the anomalous representation of the $U(n)\times U(n)$ gauge group and can be written in the form

$$W^- = \Gamma_{\mathrm{gWZW}} + W^-_{\mathrm{c}}, \tag{2.7}$$

[2] Our $\gamma_4 = \gamma_0$(Salcedo) and we made the sign choice η_4(Salcedo) $= +1$.

where $\Gamma_{\rm gWZW}$ is an extended gauged Wess-Zumino-Witten (WZW) action that contains the chiral anomaly. The remainder $W_{\rm c}^-$ is $U(n)\times U(n)$ gauge-invariant. When we specialize the gauge fields to those of the Standard Model, for which the fermion content is an anomaly-free representation of the $U(1)\times SU(2)\times SU(3)$ gauge group [27, 28, 29], $\Gamma_{\rm gWZW}$ becomes also gauge invariant (appendix B).

We start with $W_{\rm c}^-$. Using an elegant and powerful notation [25, 26] Salcedo obtains $W_{\rm c}^-$ in the condensed form

$$W_{\rm c}^-[v,m] = \frac{1}{48\pi^2}\int d^4x\, \epsilon_{\kappa\lambda\mu\nu}{\rm tr}\left[N_{123}\hat{\mathsf{D}}_\kappa \mathsf{m}\, \hat{\mathsf{D}}_\lambda \mathsf{m}\, \mathsf{F}_{\mu\nu} + N_{1234}\hat{\mathsf{D}}_\kappa \mathsf{m}\, \hat{\mathsf{D}}_\lambda \mathsf{m}\, \hat{\mathsf{D}}_\mu \mathsf{m}\, \hat{\mathsf{D}}_\nu \mathsf{m}\right]. \quad (2.8)$$

Here N_{123}, is a function of m_1, m_2 and m_3, and similarly for N_{1234}, in which the subscripts indicate the position where the matrices m are to be inserted in the trace; m and $\mathsf{F}_{\mu\nu}$ are to be replaced by $m_{\rm LR}$ or $m_{\rm RL}$ and $F^{\rm R}_{\mu\nu}$ or $F^{\rm L}_{\mu\nu}$, with $\hat{\mathsf{D}}$ the appropriate covariant derivative, such that a gauge-invariant expression results. See [25, 26] for a full exposition of the notation. To see how this works, consider the first term in (2.8) with N_{123} replaced by the monomial $\mathsf{m}_1\, \mathsf{m}_2^2\, \mathsf{m}_3^3$:

$$\begin{aligned}{\rm tr}\left[\mathsf{m}_1\, \mathsf{m}_2^2\, \mathsf{m}_3^3\, \hat{\mathsf{D}}_\kappa \mathsf{m}\, \hat{\mathsf{D}}_\lambda \mathsf{m}\, \mathsf{F}_{\mu\nu}\right] &\equiv {\rm tr}\left[\mathsf{m}\, \hat{\mathsf{D}}_\kappa \mathsf{m}\, \mathsf{m}^2\, \hat{\mathsf{D}}_\lambda \mathsf{m}\, \mathsf{m}^3\, \mathsf{F}_{\mu\nu}\right]\\ &\equiv \frac{1}{2}{\rm tr}\left[m_{\rm RL}\hat{D}_\kappa m_{\rm LR}\, m_{\rm RL} m_{\rm LR}\, \hat{D}_\lambda m_{\rm RL}\, m_{\rm LR} m_{\rm RL} m_{\rm LR}\, F^{\rm R}_{\mu\nu}\right]\\ &\quad -({\rm L}\leftrightarrow{\rm R}).\end{aligned} \quad (2.9)$$

More general functions N_{123} are dealt with by going to a basis in which $m_{\rm LR}$ and $m_{\rm RL}$ reduce to positive diagonal matrices d. This can be achieved by making a 'polar decomposition' $m_{\rm LR} = PU$ in which U is unitary and P is hermitian and positive, and then diagonalize P, $U_{\rm L}^\dagger P U_{\rm L} = d$, or $P = U_{\rm L} d\, U_{\rm L}^\dagger$, which leads to

$$m_{\rm LR} = U_{\rm L} d\, U_{\rm R}^\dagger, \quad m_{\rm RL} = U_{\rm R} d\, U_{\rm L}^\dagger, \quad (2.10)$$

with $U_{\rm R}^\dagger = U_{\rm L}^\dagger U$. We also have

$$m_{\rm LR} m_{\rm RL} = U_{\rm L} d^2\, U_{\rm L}^\dagger, \quad m_{\rm RL} m_{\rm LR} = U_{\rm R} d^2\, U_{\rm R}^\dagger, \quad (2.11)$$

etc. Even factors of m have identical L or R labels left and right. It follows that (2.9) can be written in the form

$$\begin{aligned}&\frac{1}{2}{\rm tr}\left[d\, U_{\rm L}^\dagger \hat{D}_\kappa m_{\rm LR} U_{\rm R}\, d^2\, U_{\rm R}^\dagger \hat{D}_\lambda m_{\rm RL} U_{\rm L}\, d^3\, U_{\rm R}^\dagger F^{\rm R}_{\mu\nu} U_{\rm R}\right] - ({\rm L}\leftrightarrow{\rm R})\\ &= \frac{1}{2}\sum_{jkl} d_j d_k^2 d_l^3 (\hat{D}_\kappa m_{\rm LR})_{jk} (\hat{D}_\lambda m_{\rm RL})_{kl} (F^{\rm R}_{\mu\nu})_{lj} - ({\rm L}\leftrightarrow{\rm R}),\end{aligned} \quad (2.12)$$

where $j,k,l = 1,\cdots,n$ are labels in the diagonal basis, $(d)_{jk} = d_j\delta_{jk}$, and

$$(F^{\rm R}_{\mu\nu})_{lj} = (U_{\rm R}^\dagger F^{\rm R}_{\mu\nu} U_{\rm R})_{lj}, \quad (\hat{D}_\kappa m_{\rm LR})_{jk} = (U_{\rm L}^\dagger \hat{D}_\kappa m_{\rm LR} U_{\rm R})_{jk}, \quad (\hat{D}_\lambda m_{\rm RL})_{kl} = (U_{\rm R}^\dagger \hat{D}_\lambda m_{\rm RL} U_{\rm L})_{kl}. \quad (2.13)$$

Since even factors of m do not change L into R or R into L, (2.9), ..., (2.12) can be generalized to

$$\mathrm{tr}\left[m_1^p\, m_2^q\, m_3^r\, \hat{D}_\kappa m\, \hat{D}_\lambda m\, F_{\mu\nu}\right] = \frac{1}{2}\sum_{jkl} d_j^p d_k^q d_l^r (\hat{D}_\kappa m_{\mathrm{LR}})_{jk}(\hat{D}_\lambda m_{\mathrm{RL}})_{kl}(F^{\mathrm{R}}_{\mu\nu})_{lj} - (\mathrm{L}\leftrightarrow\mathrm{R}), \tag{2.14}$$

provided that the integers p and r are odd and q is even.

The function N_{123} is given in [26] and we have copied it into appendix A. It is invariant under the simultaneous sign flips $m_a \to -m_a$, $a = 1, 2, 3$. We decompose it into terms even and odd in m_1, ..., m_3:

$$N_{123} = f^{(0)} + f^{(12)} m_1 m_2 + f^{(23)} m_2 m_3 + f^{(13)} m_1 m_3 \tag{2.15}$$

$$\equiv N^{(0)} + N^{(12)} + N^{(23)} + N^{(13)}, \tag{2.16}$$

where the fs are even functions, e.g. $N^{(12)}(m_1, m_2, m_3) = f^{(12)}(m_1^2, m_2^2, m_3^2) m_1 m_2$. The first term in the trace in (2.8) can then be written in the more explicit form

$$\begin{aligned}\mathrm{tr}\left[N_{123}\hat{D}_\kappa m\, \hat{D}_\lambda m\, F_{\mu\nu}\right] = \frac{1}{2}\sum_{jkl} \Big[& N^{(0)}_{jkl}(\hat{D}_\kappa m_{\mathrm{RL}})_{jk}(\hat{D}_\lambda m_{\mathrm{LR}})_{kl}(F^{\mathrm{R}}_{\mu\nu})_{lj}\\ &+ N^{(12)}_{jkl}(\hat{D}_\kappa m_{\mathrm{LR}})_{jk}(\hat{D}_\lambda m_{\mathrm{LR}})_{kl}(F^{\mathrm{R}}_{\mu\nu})_{lj}\\ &+ N^{(23)}_{jkl}(\hat{D}_\kappa m_{\mathrm{RL}})_{jk}(\hat{D}_\lambda m_{\mathrm{RL}})_{kl}(F^{\mathrm{R}}_{\mu\nu})_{lj}\\ &+ N^{(13)}_{jkl}(\hat{D}_\kappa m_{\mathrm{LR}})_{jk}(\hat{D}_\lambda m_{\mathrm{RL}})_{kl}(F^{\mathrm{R}}_{\mu\nu})_{lj}\Big] - (\mathrm{L}\leftrightarrow\mathrm{R}),\end{aligned} \tag{2.17}$$

where

$$N^{(12)}_{jkl} = N^{(12)}(d_j, d_k, d_l), \tag{2.18}$$

etc. The second term involving N_{1234} can be treated in similar fashion but we shall postpone this for later.

3. Application to the Standard Model

We write the fermion part of the SM action, extended with right-handed neutrino fields, in the form

$$S_{\mathrm{F}} = \int d^4x\, \bar{\Psi}\left\{\gamma_\mu\left[\partial_\mu - iA_\mu P_{\mathrm{L}} - iG_\mu - i(Y_{\mathrm{L}}P_{\mathrm{L}} + Y_{\mathrm{R}}P_{\mathrm{R}})B_\mu\right] + \Phi\Lambda P_{\mathrm{R}} + \Lambda^\dagger\Phi^\dagger P_{\mathrm{L}}\right\}\Psi. \tag{3.1}$$

Here Ψ is a 4(Dirac) $\times$ n-component spinor, where n = 2(isospin) $\times$ (3(color) + 1) $\times$ 3(family) = 24 (the leptons are represented by the '1'). The gauge fields are taken to be hermitian: B_μ for $U(1)$, A_μ for $SU(2)$ and G_μ for $SU(3)$. The matrix fields A_μ and G_μ and also the coupling matrices Y and Λ are embedded into the grand structure in the usual tensor product fashion: the spinor field has components (suppressing the Dirac indices)

$$\Psi^k, \quad k = (i, c, f), \quad i \in \{u, d\}, \quad c \in \{1, 2, 3\}, \quad f \in \{1, 2, 3\}, \tag{3.2}$$

with (weak) isospin index i, color index c and family index f, for quarks, and of course no color index for the leptons. The $SU(2)$ gauge fields can be written in terms of Pauli matrices as $A_\mu = A_\mu^a \tau_a/2$, with $(\tau_a)_{kk'} = (\tau_a)_{ii'}\delta_{cc'}\delta_{ff'}$. Similarly the $SU(3)$ fields are embedded as $(G_\mu)_{kk'} = (G_\mu)_{cc'}\delta_{ii'}\delta_{ff'}$. We find it convenient to make the $SU(2)$ structure explicit:

$$A_\mu = \frac{1}{2}\,\tau_a\,A_\mu^a, \quad \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3.3}$$

The Y are diagonal matrices representing the $U(1)$ hypercharges:

$$Y_{\mathrm{L}} = \begin{pmatrix} \frac{1}{6} & 0 \\ 0 & \frac{1}{6} \end{pmatrix} \pi_q + \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \pi_\ell, \quad Y_{\mathrm{R}} = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} \pi_q + \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \pi_\ell, \tag{3.4}$$

where π_q and π_ℓ project respectively onto the quark and lepton labels. The Higgs field is in matrix form. In terms of the $SU(2)$ Higgs-doublet $(\varphi^u, \varphi^d)^T$ it reads

$$\Phi = \begin{pmatrix} \varphi^{d*} & \varphi^u \\ -\varphi^{u*} & \varphi^d \end{pmatrix}, \quad \Phi \to \Omega\Phi e^{-i\omega\tau_3/2}, \tag{3.5}$$

where we have also indicated its behavior under gauge transformations, $e^{i\omega/6} \in U(1)$, $\Omega \in SU(2)$. Note that, since $\tau_3/2 = Y_{\mathrm{R}} - Y_{\mathrm{L}}$ and Y_{L} commutes with Φ, this can also be written as $\Phi \to \Omega_{\mathrm{L}}\Phi\Omega_{\mathrm{R}}^\dagger$, with $\Omega_{\mathrm{L}} = e^{i\omega Y_{\mathrm{L}}}\Omega$, $\Omega_{\mathrm{R}} = e^{i\omega Y_{\mathrm{R}}}$. The matrix Λ represents the Yukawa couplings. Its $SU(2)$ structure is given by

$$\Lambda = \begin{pmatrix} \Lambda_q^u & 0 \\ 0 & \Lambda_q^d \end{pmatrix} \pi_q + \begin{pmatrix} \Lambda_\ell^u & 0 \\ 0 & \Lambda_\ell^d \end{pmatrix} \pi_\ell, \tag{3.6}$$

where the $\Lambda_q^u, \ldots, \Lambda_\ell^d$ are non-trivial matrices in family space. Note that we have not included a Majorana mass term for the right-handed neutrino fields (often invoked for the see-saw mechanism), since this does not fit straight-away into the $\bar\psi \cdots \psi$ form assumed in Salcedo's calculation of the effective action.

It follows that the fields in the previous section are realized as

$$v_\mu^{\mathrm{L}} = -iY_{\mathrm{L}}B_\mu - iA_\mu - iG_\mu, \quad v_\mu^{\mathrm{R}} = -iY_{\mathrm{R}}B_\mu - iG_\mu, \quad m_{\mathrm{LR}} = \Phi\Lambda, \quad m_{\mathrm{RL}} = \Lambda^\dagger\Phi^\dagger, \tag{3.7}$$

and

$$F_{\mu\nu}^{\mathrm{L}} = -iA_{\mu\nu} - iY_{\mathrm{L}}B_{\mu\nu} - iG_{\mu\nu}, \quad F_{\mu\nu}^{\mathrm{R}} = -iY_{\mathrm{R}}B_{\mu\nu} - iG_{\mu\nu}, \tag{3.8}$$

$$\hat D_\mu m_{\mathrm{LR}} = \left(\partial_\mu\Phi - iA_\mu\Phi + i\Phi\frac{1}{2}\tau_3 B_\mu\right)\Lambda, \quad \hat D_\mu m_{\mathrm{RL}} = \left(\hat D_\mu m_{\mathrm{LR}}\right)^\dagger, \tag{3.9}$$

with $A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$, etc. The diagonal basis used in (2.17) is obtained by diagonalizing Λ and futhermore transforming to the unitary gauge in which $\Phi = h\,\mathbb{1}$ ($\sqrt{2}h$ is the standard-normalized Higgs field):

$$\Lambda = V_{\mathrm{L}}\lambda V_{\mathrm{R}}^\dagger, \quad \lambda = \begin{pmatrix} \lambda_q^u & 0 \\ 0 & \lambda_q^d \end{pmatrix} \pi_q + \begin{pmatrix} \lambda_\ell^u & 0 \\ 0 & \lambda_\ell^d \end{pmatrix} \pi_\ell, \tag{3.10}$$

$$V_{\mathrm{L}} = \begin{pmatrix} V_{\mathrm{L}}^u & 0 \\ 0 & V_{\mathrm{L}}^d \end{pmatrix}_q \pi_q + \begin{pmatrix} V_{\mathrm{L}}^u & 0 \\ 0 & V_{\mathrm{L}}^d \end{pmatrix}_\ell \pi_\ell, \tag{3.11}$$

and similar for $V_{\rm R}$; furthermore

$$\Phi = \Omega h, \quad \Omega \in SU(2), \tag{3.12}$$

and so

$$m_{\rm LR} = U_{\rm L}\, d\, U_{\rm R}^\dagger, \quad U_{\rm L} = \Omega V_{\rm L}, \quad U_{\rm R} = V_{\rm R}, \quad d = h\,\lambda. \tag{3.13}$$

The λ are non-trivial diagonal matrices in family space, $\lambda_q^u = {\rm diag}(\lambda_u, \lambda_c, \lambda_t)$, $\lambda_q^d = {\rm diag}(\lambda_d, \lambda_s, \lambda_b)$, and similar for the leptons. In the following we will concentrate on the quark contribution to the effective action (the lepton contribution is analogous), and omit the subscripts q and ℓ if there is no danger of confusion.

The matrix elements of the covariant derivatives entering in (2.17) in the diagonal basis are now given by

$$\begin{aligned} U_{\rm L}^\dagger \hat{D}_\mu m_{\rm LR} U_{\rm R} &= V_{\rm L}^\dagger (h^{-1}\partial_\mu h - iW_\mu + iB_\mu \tau_3/2) V_{\rm L} d & (3.14)\\ &\equiv -iC_\mu d, & (3.15)\\ U_{\rm R}^\dagger \hat{D}_\mu m_{\rm RL} U_{\rm L} &= id\, C_\mu^\dagger, & (3.16)\\ C_\mu &= ih^{-1}\partial_\mu h + W_\mu^a \tilde{\tau}_a/2 - B_\mu \tau_3/2, & (3.17)\end{aligned}$$

where W_μ is the $SU(2)$ gauge field in unitary gauge,

$$W_\mu = \Omega^\dagger A_\mu \Omega + i\Omega^\dagger \partial_\mu \Omega, \tag{3.18}$$

and

$$\tilde{\tau}_1 = \begin{pmatrix} 0 & V \\ V^\dagger & 0 \end{pmatrix}, \quad \tilde{\tau}_2 = \begin{pmatrix} 0 & -iV \\ iV^\dagger & 0 \end{pmatrix}, \quad \tilde{\tau}_3 = \tau_3, \quad V = V_{\rm L}^{u\dagger} V_{\rm L}^d. \tag{3.19}$$

Here V is the celebrated CKM matrix. Similarly,

$$\begin{aligned} U_{\rm L}^\dagger F_{\mu\nu}^{\rm L} U_{\rm L} &= -i\tilde{W}_{\mu\nu} - iY_{\rm L} B_{\mu\nu} - iG_{\mu\nu}, & (3.20)\\ U_{\rm R}^\dagger F_{\mu\nu}^{\rm R} U_{\rm R} &= -iY_{\rm R} B_{\mu\nu} - iG_{\mu\nu} = F_{\mu\nu}^{\rm R}, & (3.21)\end{aligned}$$

with

$$\tilde{W}_{\mu\nu} = W_{\mu\nu}^a \tilde{\tau}_a/2. \tag{3.22}$$

Note that in (3.17) the combination $W_\mu^3 - B_\mu = Z_\mu$, the Z field with coupling constants absorbed,

$$B_\mu = A_\mu - \sin^2\theta_{\rm W} Z_\mu, \quad W_\mu^3 = A_\mu + \cos^2\theta_{\rm W} Z_\mu, \tag{3.23}$$

with A_μ is the photon field (with electro-magnetic coupling e absorbed) and $\theta_{\rm W}$ the Weinberg angle.

Consider now the $N^{(0)}$ contribution in (2.17). With the above specialization to the SM this becomes, including also the epsilon tensor from (2.8),

$$\begin{aligned} &\epsilon_{\kappa\lambda\mu\nu} \frac{1}{2} \sum_{jkl} \Big[N_{jkl}^{(0)}\, d_j (C_\kappa^\dagger)_{jk} (C_\lambda)_{kl}\, d_l\, (-iY_{\rm R} B_{\mu\nu} - iG_{\mu\nu})_{lj} \\ &\quad - N_{jkl}^{(0)} (C_\kappa)_{jk}\, d_k^2\, (C_\lambda^\dagger)_{kl}\, (-i\tilde{W}_{\mu\nu} - iY_{\rm L} B_{\mu\nu} - iG_{\mu\nu})_{lj} \Big]. \end{aligned} \tag{3.24}$$

Firstly, we observe that the $SU(3)$ fields do not contribute, since the egenvalues d_j and hence also $N^{(0)}_{jkl}$ are color-independent, $(C_\mu)_{kk'} \propto \delta_{cc'}$ (cf. (3.2)), and $G_{\mu\nu}$ is traceless. Secondly, because of the property (cf. (A.4))

$$N^{(0)}_{jkl} = -N^{(0)}_{lkj}, \tag{3.25}$$

we have $N^{(0)}_{jkj} = 0$, and consequently also the fields $B_{\mu\nu}$ and $W^3_{\mu\nu}$ involving the diagonal generators $Y_{\mathrm{L,R}}$ and τ_3 drop out. So we are left with the off-diagonal contribution from $\tilde{W}_{\mu\nu}$:

$$\epsilon_{\kappa\lambda\mu\nu}\frac{i}{4}\sum_{jkl} N^{(0)}_{jkl} d_k^2\, (C_\kappa)_{jk}\, (C^\dagger_\lambda)_{kl}\, (\tilde{\tau}_1 W^1_{\mu\nu} + \tilde{\tau}_2 W^2_{\mu\nu})_{lj}. \tag{3.26}$$

Using interchange of dummy indices $\kappa \leftrightarrow \lambda$ this can also be written as

$$\epsilon_{\kappa\lambda\mu\nu}\frac{-i}{4}\sum_{jkl} N^{(0)}_{jkl} d_k^2\, (C_\lambda)_{jk}\, (C^\dagger_\kappa)_{kl}\, (\tilde{\tau}_1 W^1_{\mu\nu} + \tilde{\tau}_2 W^2_{\mu\nu})_{lj}, \tag{3.27}$$

and using furthermore $j \leftrightarrow l$, the property (3.25) and the hermiticity of the $\tilde{\tau}_a$, this can be rewritten as

$$\epsilon_{\kappa\lambda\mu\nu}\frac{i}{4}\sum_{jkl} N^{(0)}_{jkl} d_k^2\, (C^*_\kappa)_{jk}\, (C^{\dagger *}_\lambda)_{kl}\, (\tilde{\tau}^*_1 W^1_{\mu\nu} + \tilde{\tau}^*_2 W^2_{\mu\nu})_{lj}. \tag{3.28}$$

Combining (3.26) and (3.28), it follows that the expression is purely imaginary,

$$\epsilon_{\kappa\lambda\mu\nu}\frac{i}{4}\sum_{jkl} N^{(0)}_{jkl} d_k^2\, \mathrm{Re}\left[(C_\kappa)_{jk}\, (C^\dagger_\lambda)_{kl}\, (\tilde{\tau}_1 W^1_{\mu\nu} + \tilde{\tau}_2 W^2_{\mu\nu})_{lj}\right], \tag{3.29}$$

which is a general property of the pseudoparity-odd contribution to the euclidean effective action. We can now examine the type of contributions:

(*i*) $C_\kappa \to W^a_\kappa \tilde{\tau}_a/2$, $C_\lambda \to W^b_\lambda \tilde{\tau}_b/2$ leads to

$$i\epsilon_{\kappa\lambda\mu\nu} W^a_\kappa\, W^b_\lambda\, W^c_{\mu\nu}\, n^{(0)}_{abc}, \tag{3.30}$$

with

$$n^{(0)}_{abc} = \frac{1}{16}\sum_{jkl} N^{(0)}_{jkl} d_k^2\, \mathrm{Re}\left[(\tilde{\tau}_a)_{jk}\, (\tilde{\tau}_b)_{kl}\, (\tilde{\tau}_c)_{lj}\right]. \tag{3.31}$$

We need to investigate $n^{(0)}_{abc}$. As seen above, it is nonzero only for $c = 1, 2$. Suppose $a = 3$. Then b has to be 1 or 2 because of the off-diagonality of $\tilde{\tau}_c$. Similarly, if $b = 3$ then only $a = 1, 2$ can give a non-zero contribution. The two cases $a = 3$ or $b = 3$ lead essentially to the same result and we continue with $a = 3$. In this case only $j = k$ contributes because τ_3 is diagonal. In the notation (3.2), let $k = (i, c, f)$, $l = (i', c', g)$. The matrices $\tilde{\tau}_a$ are color-diagonal, so we only need $c = c'$. The eigenvalues d_j do not depend on color, $d_k = d_{if}$, and the summation over c just gives a factor 3. For $b = c = 1$ this gives

$$n^{(0)}_{311} = \frac{3}{16}\sum_{fg}\left[N^{(0)}_{uf,uf,dg} d^2_{uf} \mathrm{Re}\left(V_{fg} V^\dagger_{gf}\right) - N^{(0)}_{df,df,ug} d^2_{df} \mathrm{Re}\left(V^\dagger_{fg} V_{gf}\right)\right], \tag{3.32}$$

and the same for $b = c = 2$, leading to a contribution

$$i\epsilon_{\kappa\lambda\mu\nu} W^3_\kappa \left(W^1_\lambda W^1_{\mu\nu} + W^2_\lambda W^2_{\mu\nu} \right) n^{(0)}_{311}. \tag{3.33}$$

For $b = 1$ and $c = 2$ we get

$$\begin{aligned} n^{(0)}_{312} &= -\frac{3}{16} \sum_{fg} \left[N^{(0)}_{uf,uf,dg} d^2_{uf} \mathrm{Im} \left(V_{fg} V^\dagger_{gf} \right) + N^{(0)}_{df,df,ug} d^2_{df} \mathrm{Im} \left(V^\dagger_{fg} V_{gf} \right) \right] \\ &= 0, \end{aligned} \tag{3.34}$$

since the imaginary part is zero, and 'minus zero' for $b = 2$, $c = 1$, i.e. the coefficient of

$$i\epsilon_{\kappa\lambda\mu\nu} W^3_\kappa (W^1_\lambda W^2_{\mu\nu} - W^2_\lambda W^1_{\mu\nu}) \tag{3.35}$$

vanishes.

(*ii*) $C_\kappa \to ih^{-1}\partial_\kappa h$, $C_\lambda \to W^b_\lambda \tilde{\tau}_b/2$ leads to

$$i\epsilon_{\kappa\lambda\mu\nu} h^{-1} \partial_\kappa h \, W^b_\lambda W^c_{\mu\nu} \, n^{(0)}_{bc}, \tag{3.36}$$

with

$$n^{(0)}_{bc} = -\frac{1}{8} \sum_{jkl} N^{(0)}_{jkl} d^2_k \, \delta_{jk} \, \mathrm{Im} \left[(\tilde{\tau}_b)_{kl} \, (\tilde{\tau}_c)_{lj} \right]. \tag{3.37}$$

In this case we find non-zero results only for $b = 1$, $c = 2$ and $b = 2$, $c = 1$.

The examples above show the general feature that also holds for the other contributions involving $N^{(12)}$, $N^{(23)}$ and $N^{(13)}$ in (2.17): CP conserving terms such as (3.33), and (3.36) with $b \neq c = 1, 2$, survive, but all the CP-violating contributions like (3.35) vanish.[3] The reason is evidently that there are not enough CKM matrices present in the above expressions to be able to construct even the minimal CP-violating invariant under phase redefinitions, (1.3), which is of fourth order in V, V^*.

It is now also not difficult to see that the N_{1234} contribution in (2.8) cannot contain CP-violating terms in the Standard Model case. For example, the $N^{(0)}_{jklm}$ contribution leads to terms of the form

$$\epsilon_{\kappa\lambda\mu\nu} \sum_{jklm} N^{(0)}_{jklm} d^2_j \, d^2_l \, (C^\dagger_\kappa)_{jk} (C_\lambda)_{kl} (C^\dagger_\mu)_{lm} (C_\nu)_{mj} \tag{3.38}$$

Using $N^{(0)}_{jklm} = -N^{(0)}_{lkjm}$, which follows from the properties (A.10) and the fact that $N^{(0)}_{jklm}$ is an even function of d_j, $\ldots$, d_m, this expression can be shown to be purely imaginary. Choosing from the C the purely gauge-field contribution leads to

$$i\epsilon_{\kappa\lambda\mu\nu} W^a_\kappa W^b_\lambda W^c_\mu W^d_\nu \frac{1}{16} \sum_{jklm} N^{(0)}_{jklm} d^2_j \, d^2_l \, \mathrm{Im} \left[(\tilde{\tau}_a)_{jk} (\tilde{\tau}_b)_{kl} (\tilde{\tau}_c)_{lm} (\tilde{\tau}_d)_{mj} \right], \tag{3.39}$$

[3] Under CP the fields transform as $\Phi(x) \to \Phi^*(Px)$, $A_\mu(x) \to -P_{\mu\nu} A^T_\mu(Px)$, $B_\mu(x) \to -P_{\mu\nu} B_\nu(Px)$, with $P = \mathrm{diag}(1, -1, -1, -1)$. Specifically in unitary gauge, $h(x) \to h(Px)$, $W^{1,3}_\mu \to -P_{\mu\nu} W^{1,3}_\nu(Px)$, $W^2_\mu(x) \to +P_{\mu\nu} W^2_\nu(Px)$, $Z_\mu(x) \to -P_{\mu\nu} Z_\nu(Px)$ and $A_\mu(x) \to -P_{\mu\nu} A_\nu(Px)$.

where the W^3 field can be replaced by the Z field. The above expression appears to contain enough factors of V and $V^\dagger$ to be able to make up the invariant J. However, the ϵ-tensor projects this contribution to zero as there are not enough independent four-vectors. Next, assume the Higgs field contribution in one of the C, say $C_\nu \to ih^{-1}\partial_\nu h$, which leads to

$$i\epsilon_{\kappa\lambda\mu\nu} W^a_\kappa W^b_\lambda W^c_\mu \, h^{-1}\partial_\nu h \, \frac{1}{8} \sum_{jklj} N^{(0)}_{jklm} d_j^2 \, d_l^2 \, \mathrm{Re}\left[(\tilde{\tau}_a)_{jk}(\tilde{\tau}_b)_{kl}(\tilde{\tau}_c)_{lm}\right], \tag{3.40}$$

which violates CP. The ϵ-tensor requires a, b, c to be a permutation of 1,2,3. Since $\tilde{\tau}_3$ does not contain V and τ_2 is imaginary this implies taking the imaginary part of a phase-invariant combination of two only V's, which is zero.

We conclude that to fourth order in the derivative expansion, there are no CP-violating terms in W_{c}^-. A similar analysis and conclusion applies to the analog mixing matrix in the lepton contribution to the W_{c}^-, provided there is no Majorana neutrino mass term.

4. Magnitude of CP violation

To find CP violation coming from the CKM matrix we need to go to higher order in the derivative expansion. For example, we anticipate in W_{c}^- a sixth-order term of the form

$$\epsilon_{\kappa\lambda\, u\nu} \mathrm{tr}\left[N'_{1234} F_{\kappa\lambda} F_{\mu\nu} \hat{D}_\rho m \, \hat{D}_\rho m\right]. \tag{4.1}$$

Decomposing as before $N'_{1234} = N'^{(0)}_{1234} + \cdots$, in which $N'^{(0)}_{1234}$ depends only on m^2, this contains the contribution

$$\frac{-i}{2} \sum_{jklm} N'^{(0)}_{jklm} \mathrm{Im}\left[(F^{\mathrm{L}}_{\kappa\lambda})_{jk}(F^{\mathrm{L}}_{\mu\nu})_{kl}(\hat{D}_\rho m_{\mathrm{LR}})_{lm}(\hat{D}_\rho m_{\mathrm{RL}})_{mj}\right] - (\mathrm{L} \leftrightarrow \mathrm{R}), \tag{4.2}$$

where we also used the fact that W^- is imaginary (assuming the N-functions to be real as for the fourth-order terms). The purely $SU(2)$-field contribution is then given by

$$\frac{-i}{32} \epsilon_{\kappa\lambda\mu\nu} W^a_{\kappa\lambda} W^b_{\mu\nu} W^c_\rho W^d_\rho \sum_{jklm} N'^{(0)}_{jklm} \, d_m^2 \, \mathrm{Im}\left[(\tilde{\tau}_a)_{jk}(\tilde{\tau}_b)_{kl}(\tilde{\tau}_c)_{lm}(\tilde{\tau}_d)_{mj}\right]. \tag{4.3}$$

There are several CP-violating contributions, e.g. the ones with $a = b = 1, 2$, $c = d = 1, 2$, are proportional to

$$\sum_{f,g,h,i} \left\{N'^{(0)}_{uf,dg,uh,di} \, d_{di}^2 \, \mathrm{Im}\left[V_{fg} V^*_{hg} V_{hi} V^*_{fi}\right] + N'^{(0)}_{df,ug,dh,ui} \, d_{ui}^2 \, \mathrm{Im}\left[V^*_{gf} V_{gh} V^*_{ih} V_{if}\right]\right\}, \tag{4.4}$$

in which the expected rephasing invariant J appears. Generically we do not expect these contributions to vanish. Their explicit calculation appears a very complicated task. However, we now argue that they are *not* accompanied by the tiny product of Yukawa couplings in (1.4).

A striking feature of the N_{123} and N_{1234} of the fourth-order contribution is the fact that they are homogeneous functions (cf. appendix A):

$$N(sm_1, sm_2, sm_3) = s^{-2} N(m_1, m_2, m_3), \quad N(sm_1, sm_2, sm_3, sm_4) = s^{-4} N(m_1, m_2, m_3, m_4). \tag{4.5}$$

Recalling (cf. 3.13) that the eigenvalues of m are given by $d_j = h\lambda_j$, it follows that in expressions such as (3.24) or (3.32) (recall also that $N^{(0)}_{jkl} = N^{(0)}(d_j, d_k, d_l)$, as in (2.18)), the Higgs field h hidden in d drops out altogether. It only occurs via its derivative in the combination $h^{-1}\partial_\mu h$ as in (3.36). Furthermore, the overall scale of the Yukawa couplings does not matter: rescaling $\lambda_j \to s\lambda_j$ does not change these expressions. This can be seen clearly from the explicit expression for the combination $N^{(0)}_{jkl} d_k^2$ appearing e.g. in (3.24), (3.32) and (3.37): in terms of $d_j \equiv x$, $d_k \equiv y$, $d_l \equiv z$ it reads (cf. A.6))

$$N^{(0)}_{jkl} d_k^2 = 2\left\{\frac{2x^4 + 2z^4 - 2x^2z^2 - x^2y^2 - z^2y^2}{(x^2 - y^2)(z^2 - y^2)(x^2 - z^2)}\right. \tag{4.6}$$

$$+\left[\frac{x^4y^2 + z^4y^2 + x^2z^4 - 3x^4z^2}{(x^2 - y^2)^2(x^2 - z^2)^2}\log\frac{x^2}{y^2}\right. \tag{4.7}$$

$$\left.\left. -(x \leftrightarrow z)\cdot\right]\right\} y^2. \tag{4.8}$$

Hence, the fourth-order contribution to W^- is invariant under $\lambda_j \to s\lambda_j$. Such an insensitivity to the overall scale of the λ's may very well be present also in the CP-violating terms in higher orders of the derivative expansion, such as anticipated in (4.4). This strongly suggests that the product of λ's should be ignored in rough estimates of the magnitude of CP violation. For example,

$$\frac{(\lambda_u^2 - \lambda_c^2)(\lambda_c^2 - \lambda_t^2)(\lambda_t^2 - \lambda_u^2)(\lambda_d^2 - \lambda_s^2)(\lambda_s^2 - \lambda_b^2)(\lambda_b^2 - \lambda_d^2)}{(\lambda_u^2 + \lambda_c^2)(\lambda_c^2 + \lambda_t^2)(\lambda_t^2 + \lambda_u^2)(\lambda_d^2 + \lambda_s^2)(\lambda_s^2 + \lambda_b^2)(\lambda_b^2 + \lambda_d^2)} \simeq 0.99. \tag{4.9}$$

The reasoning above does not apply to the case of finite temperature T, for which T provides a new scale. Salcedo's results used here hold only for zero temperature. For example, at sufficiently high temperature we may expect the appearance of hard-thermal-loop masses $m_{\rm th}$, via $\lambda_j h \to \lambda_j h + m^j_{\rm th}$. For quarks the QCD contribution dominates, $m^j_{\rm th} \approx g_{\rm s}T/\sqrt{6}$, with $g_{\rm s}$ the strong ($SU(3)$) gauge coupling (see e.g. [30]). There is no reason to expect the thermal masses to cancel completely in the denominators and the finite-temperature estimate (1.2) may still hold truth.

5. The extended gauged Wess-Zumino-Witten action

We now turn to the $\Gamma_{\rm gWZW}$ part of the effective action. It is given in [26] using the notation of differential forms, in addition to the notational conventions already used in section 2. The following one-forms are introduced [26]:

$$R = m^{-1}dm, \quad L = m\,dm^{-1}. \tag{5.1}$$

In terms of these, Salcedo's extended gauged WZW action is given by

$$\Gamma_{\rm gWZW} = \frac{1}{48\pi^2}\int \mathrm{tr}\left(-\frac{1}{5}R^5\right)$$
$$+\frac{1}{48\pi^2}\int \mathrm{tr}\left[-(R^3 + L^3)v + \frac{1}{2}(Rv)^2 + \frac{1}{2}(Lv)^2 + R^2vm^{-1}vm + L^2vmvm^{-1}\right.$$

$$+Rm^{-1}vmdv + Lmvm^{-1}dv + (R+L)v^3 + Rvm^{-1}vmv + Lvmvm^{-1}v$$
$$+(R + L + m^{-1}vm + mvm^{-1})\{v, dv\} + mvm^{-1}v^3 + m^{-1}vmv^3$$
$$\left. + \frac{1}{2}(mvm^{-1}v)^2 \right] . \tag{5.2}$$

The first integral is over a five-dimensional manifold which has four-dimensional euclidean space-time as a boundary. The second integral is over four-dimensional space-time. An alternative version [26] that exhibits the properties under gauge transformations more clearly is recalled in appendix B.

Because the SM reduction is gauge invariant we may use again the unitary gauge, which makes it easier to deal with the factors of m^{-1}. Consider for example

$$\begin{aligned}\int \mathrm{tr}\,\left[Rm^{-1}vmdv\right] &= \int d^4x\, \epsilon_{\kappa\lambda\mu\nu}\, \mathrm{tr}\,\left[m^{-1}\partial_\kappa m\, m^{-1} v_\lambda m\, \partial_\mu v_\nu\right] \\ &\equiv \int d^4x\, \epsilon_{\kappa\lambda\mu\nu}\, \frac{1}{2}\,\mathrm{tr}\,\left[m_{\mathrm{LR}}^{-1}\partial_\kappa m_{\mathrm{LR}}\, m_{\mathrm{LR}}^{-1} v_\lambda^{\mathrm{L}} m_{\mathrm{LR}}\, \partial_\mu v_\nu^{\mathrm{R}}\right] - (\mathrm{L} \leftrightarrow \mathrm{R}).\end{aligned} \tag{5.3}$$

In the unitary gauge $m_{\mathrm{LR}} = h\Lambda$, $m_{\mathrm{LR}}^{-1} = h^{-1}\Lambda^{-1}$, and $m_{\mathrm{LR}}^{-1}\partial_\kappa m_{\mathrm{LR}} = m_{\mathrm{RL}}^{-1}\partial_\kappa m_{\mathrm{RL}} = h^{-1}\partial_\kappa h$. This gives (cf. (3.7))

$$\begin{aligned}&(-i)^2 \int d^4x\, \epsilon_{\kappa\lambda\mu\nu} \left\{ \frac{1}{2}\, h^{-1}\partial_\kappa h\, \mathrm{tr}\,\left[\Lambda^{-1}(G_\lambda + W_\lambda + Y_{\mathrm{L}}B_\lambda)\Lambda\, \partial_\mu(G_\nu + Y_{\mathrm{R}}B_\nu)\right] \right. \\ &\left. - \frac{1}{2}\, h^{-1}\partial_\kappa h\, \mathrm{tr}\,\left[\Lambda^{\dagger-1}(G_\lambda + Y_{\mathrm{R}}B_\lambda)\Lambda^\dagger\, \partial_\mu(G_\nu + W_\nu + Y_{\mathrm{L}}B_\nu)\right] \right\} \\ &= 0,\end{aligned} \tag{5.4}$$

where we used the fact that Y_{R} and Y_{L} commute with Λ, and partial integration. Evaluating all the terms this way we find[4]

$$\Gamma_{\mathrm{gWZW}} = 0. \tag{5.5}$$

An unsatisfactory aspect of this result is that, since total derivatives have been dropped, the QCD θ-term has been lost as well. It is supposed to be produced by the chiral anomaly, upon diagonalization of the quark mass-matrix $\propto \Lambda_q$. To recover θ terms, we initially allow Λ to be space-time dependent in the reduction to the Standard Model. Then the purely $SU(3)$ gauge-field contribution to (5.3) produces factors (cf. (3.6))

$$\mathrm{tr}_{\,\mathrm{if}}(\Lambda^{-1}\partial_\kappa\Lambda\, \pi_q) = \mathrm{tr}_{\,\mathrm{f}}(\Lambda_q^{u\,-1}\partial_\kappa\Lambda_q^u) + (u \to d) = \partial_\kappa \mathrm{tr}_{\,\mathrm{f}}(\ln\Lambda_q^u) + (u \to d) = i\partial_\kappa(\theta_q^u + \theta_q^d) \tag{5.6}$$

where $\theta_q^{u,d} = \arg\det\Lambda_q^{u,d}$ and $\mathrm{tr}_{\,\mathrm{if}}$ and $\mathrm{tr}_{\,\mathrm{f}}$ are traces in isospin-family and in family space, respectively. We assume that the $\theta \to 0$ as $|x| \to \infty$ fast enough, initially, to allow for partial integration without surface terms. After removing ∂_κ from the θ's by partial integration they are taken to be constant. We then recover the QCD θ-term with $\theta = \theta_q^u + \theta_q^d$ from the terms linear in L and R (see also below). In the complete case with also the $U(1)$ and $SU(2)$ gauge fields present there may be also be contributions from the terms non-linear in

[4] Since there are an even number of $SU(2)$ doublets, the first term in (5.2) is an unobservable multiple of $2\pi i$; it is evidently zero in the unitary gauge because of antisymmetry in the differential form.

L and R. To avoid such contributions we promote only the phase of the total determinants of Λ_q and Λ_ℓ to axion-like fields, writing

$$\Lambda_q^{-1}\partial_\kappa\Lambda_q = i\partial_\kappa\theta_q \frac{1}{n_{\rm if}}\,\mathbb{1}, \quad \theta_q = \arg\det\Lambda_q, \tag{5.7}$$

and similar for $q \to \ell$; here $n_{\rm if} = 6$ is the number of families times the dimension of isospin space. With only two independent vectors, $\partial_\kappa\theta_q$ and $\partial_\kappa\theta_\ell$, the θ can appear only linearly in $\Gamma_{\rm gWZW}$ because of $\epsilon_{\kappa\lambda\mu\nu}$, since quark and lepton contributions are not mixed. We can implement (5.7) as $\Lambda_q = \Lambda'_q\, e^{i\theta_q/n_{\rm if}}$, $\det\Lambda'_q = 1$, with Λ'_q independent of x. Effectively this implies

$$m_{\rm LR}^{-1}\partial_\kappa m_{\rm LR} \to h^{-1}\partial_\kappa h + i\partial_\kappa\theta/n_{\rm if}, \quad m_{\rm RL}^{-1}\partial_\kappa m_{\rm RL} \to h^{-1}\partial_\kappa h - i\partial_\kappa\theta/n_{\rm if}, \tag{5.8}$$

with

$$\theta = \theta_q\,\pi_q + \theta_\ell\,\pi_\ell. \tag{5.9}$$

In addition to (5.4) we now also get the non-zero contribution

$$\begin{aligned} -i\frac{1}{n_{\rm if}}\int d^4x\,\epsilon_{\kappa\lambda\mu\nu}\,\Big\{&\frac{1}{2}{\rm tr}\,\left[\partial_\kappa\theta\,(G_\lambda + W_\lambda + Y_{\rm L}B_\lambda)\partial_\mu(G_\nu + Y_{\rm R}B_\nu)\right] \\ +&\frac{1}{2}{\rm tr}\,\left[\partial_\kappa\theta\,(G_\lambda + Y_{\rm R}B_\lambda)\,\partial_\mu(G_\nu + W_\nu + Y_{\rm L}B_\nu)\right]\Big\}. \end{aligned} \tag{5.10}$$

Collecting all the terms and making a partial integration to take away the derivative from θ we get

$$\begin{aligned} \Gamma_{\rm gWZW} = i\frac{1}{48\pi^2}\int d^4x\,\epsilon_{\kappa\lambda\mu\nu}\,\partial_\kappa\frac{1}{n_{\rm if}}\,{\rm tr}\,\{&\theta[6G_\lambda\partial_\mu G_\nu - i4G_\lambda G_\mu G_\nu \\ &+ (W_\lambda + Y_{\rm L}B_\lambda)\partial_\mu B_\nu + Y_{\rm R}B_\lambda\partial_\mu(W_\nu + Y_{\rm L}B_\nu) \\ &- i(W_\lambda + Y_{\rm L}B_\lambda)(W_\mu + Y_{\rm L}B_\mu)(W_\nu + Y_{\rm L}B_\nu) \\ &- i(W_\lambda + Y_{\rm L}B_\lambda)Y_{\rm R}B_\mu(W_\nu + Y_{\rm L}B_\nu) \\ &+2Y_{\rm R}^2B_\lambda\partial_\mu B_\nu + 2(W_\lambda + Y_{\rm L}B_\lambda)\partial_\mu(W_\nu + Y_{\rm L}B_\nu)]\}. \end{aligned} \tag{5.11}$$

Except for the first line, the order of the terms corresponds roughly to the order of the terms in (5.2).

We now let $\theta_{q,\ell}$ become constants. Then the first line takes the form of the QCD θ-term, since

$$\begin{aligned} &\epsilon_{\kappa\lambda\mu\nu}\frac{1}{48\pi^2 n_{\rm if}}\,\partial_\kappa{\rm tr}\,\{\theta[6G_\lambda\partial_\mu G_\nu - i4G_\lambda G_\mu G_\nu]\} \\ &= \theta_q\,\frac{1}{8\pi^2}\,\epsilon_{\kappa\lambda\mu\nu}\,\partial_\kappa{\rm tr}_{\,\rm c}[G_\lambda\partial_\mu G_\nu - i(2/3)\,G_\lambda G_\mu G_\nu] = \theta_q\,\partial_\kappa j_\kappa^{\rm CS}, \end{aligned} \tag{5.12}$$

with $j_\kappa^{\rm CS}$ the Chern-Simons current (${\rm tr}_{\,\rm c}$ is the trace in color space). The terms involving only the $SU(2)$ gauge field are given by

$$i\int d^4x\,(n_{\rm c}\theta_q + \theta_\ell)\epsilon_{\kappa\lambda\mu\nu}\frac{1}{192\pi^2}\partial_\kappa{\rm tr}_{\,\rm i}\left(W_\lambda\partial_\mu W_\nu - i\frac{1}{2}W_\lambda W_\mu W_\nu\right) \tag{5.13}$$

where $n_{\rm c} = 3$ is the number of colors and $\mathrm{tr}_{\rm i}$ is the trace in isospin space. This expression has been derived in the unitary gauge and the integrand is not explicitly gauge-invariant anymore. The above expression is furthermore not a topological object like the divergence of a Chern-Simons current and being a total derivative it has presumably no physical significance. Even if it did have the form of a θ term, it would still not lead to observable effects according to [31, 32]. Since we are working in unitary gauge it is natural to express the remaining contribution in terms the Z-field, the charged W-fields and the photon field A_μ. The photon-field contribution is given by

$$i \int d^4x \left[n_{\rm c} \left(\frac{4}{9} + \frac{1}{9} \right) \theta_q + \theta_\ell \right] \frac{1}{64\pi^2} \, \epsilon_{\kappa\lambda\mu\nu} \, A_{\kappa\lambda} A_{\mu\nu}, \tag{5.14}$$

The integrand has topological significance in a finite four-dimensional torus [33], but unlike the QCD case it probably has no physical significance.[5] The remaining terms involving also the W and Z fields are cumbersome and not particularly illuminating. As integrals of total derivatives involving massive fields they are expected to be physically irrelevant.

6. Conclusion

Using Salcedo's results for the effective action we have shown that the CP violation in the Standard Model coming from the CKM matrix is absent to fourth order in the gauge-covariant derivative-expansion. Six or more orders in the covariant derivatives of the fields are needed for CKM-type CP violation. The same holds for the analog mixing matrix in the lepton sector, which becomes relevant upon extending the SM with Yukawa couplings such that the neutrinos are given Dirac mass terms. The possibility of Majorana mass terms in the neutrino sector is very interesting in the present context as their presence limits the rephasing invariance, perhaps allowing for a non-zero CP violating contribution to the effective action already at fourth order. We leave this question for future investigation.

With a trick of introducing axion-like fields we were able to recover the known CP-violating total-derivative QCD θ-term, and electroweak analogs which are not expected to have physical consequences (see also [31, 32]).

Last, but not least, the homogeneity of the coefficient functions calculated by Salcedo strongly suggests that we should not include the tiny ($\approx 10^{-17}$) product of Yukawa couplings (cf. (1.4)) in order-of-magnitude estimates of CKM CP-violation at zero temperature. This argument does not apply to the high-temperature case, for which (1.2) may still be of value.[6]

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[5] For example, the corresponding topological suceptibility would scale to zero like e^4/volume in the infinite-volume limit.

[6] Replacing T by the QCD thermal quark mass at temperatures above the electroweak scale we would gain a factor $(g_{\rm s}/\sqrt{6})^{-12} \gtrsim 10^4$.

A. The functions N_{123} and N_{1234}

Salcedo's function N_{123} is given by [26]

$$N_{123} = N_{123}^R + N_{123}^L \log(m_1^2/m_2^2) - N_{321}^L \log(m_3^2/m_2^2)\,, \tag{A.1}$$

with

$$\begin{aligned} N_{123}^R &= \frac{1}{2m_1 m_2 m_3 (m_1^2 - m_2^2)(m_3^2 - m_2^2)(m_1 - m_3)} \\ &\times\Big(3m_1^2 m_3^2 (m_1 - m_3)^2 + 4m_1 m_2 m_3 (m_1 + m_3)(2m_1^2 - 3m_1 m_3 + 2m_3^2 - m_2^2) \\ &\quad + m_2^2 (m_1^4 + 10 m_1^3 m_3 - 18 m_1^2 m_3^2 + 10 m_1 m_3^3 + m_3^4) - m_2^4 (m_1 + m_3)^2\Big)\,, \quad (\text{A.2}) \\ N_{123}^L &= \frac{2}{(m_1^2 - m_2^2)^2 (m_1^2 - m_3^2)(m_1 - m_3)} \\ &\times\Big(m_1^4 (m_2 - 2m_3) + m_1^2 (m_2^3 + m_3^3) + m_2^2 m_3^2 (m_2 + m_3) \\ &\quad + m_1^3 (m_2^2 - 3 m_2 m_3 - m_3^2) - m_1 m_2 m_3 (m_2^2 - m_3^2)\Big)\,. \quad (\text{A.3}) \end{aligned}$$

This function satisfies $N_{123} = N_{\underline{123}}$, $N_{123} = -N_{321}$, where $N_{\ldots\underline{j}\ldots} \equiv N(m_j \to -m_j)$; explicitly:

$$N(m_1, m_2, m_3) = N(-m_1, -m_2, -m_3), \quad N(m_1, m_2, m_3) = -N(m_3, m_2, m_1), \tag{A.4}$$

It is furthermore regular at coinciding arguments. The functions $N_{123}^{(\cdot)}$ introduced in section 2 are found by taking appropriate even-odd combinations. For example, $N_{123}^{(12)}$ is defined by

$$N_{123}^{(12)} = \frac{1}{4}\left[N(m_1, m_2, m_3) - N(-m_1, m_2, m_3) - N(m_1, -m_2, m_3) + N(-m_1, -m_2, m_3)\right], \tag{A.5}$$

and similar for $N_{123}^{(23)}$ and $N_{123}^{(13)}$, whereas

$$\begin{aligned} N_{123}^{(0)} = \frac{1}{8}\big[&N(m_1, m_2, m_3) + N(-m_1, m_2, m_3) + N(m_1, -m_2, m_3) + N(-m_1, -m_2, m_3) \quad (\text{A.6}) \\ &+ N(m_1, m_2, -m_3) + N(-m_1, m_2, -m_3) + N(m_1, -m_2, -m_3) + N(-m_1, -m_2, -m_3)\big] \end{aligned}$$

It follows that $N^{(0)}$ and $f^{(pq)} = N^{(pq)}/m_p m_q$ are functions of m_1^2, m_2^2 and m_3^2. The function N_{1234} is given by [26]

$$N_{1234} = N_{1234}^R + N_{1234}^L \log(m_1^2) + N_{234\underline{1}}^L \log(m_2^2) + N_{34\underline{12}}^L \log(m_3^2) + N_{4\underline{123}}^L \log(m_4^2)\,, \tag{A.7}$$

where

$$\begin{aligned} N_{1234}^R = \frac{1}{4}\Big(&\frac{2(2m_2 + m_3)}{(m_1^2 - m_2^2)(m_2^2 - m_3^2)(m_2 - m_4)} - \frac{2(2m_2 + m_1)}{(m_1^2 - m_2^2)(m_2^2 - m_3^2)(m_2 + m_4)} \\ &- \frac{3(m_2 m_3 - m_1(m_2 + m_3))}{m_3(m_1^2 - m_3^2)(m_2^2 - m_3^2)(m_3 - m_4)} + \frac{3(m_1 m_2 - m_3(m_1 + m_2))}{m_1(m_1^2 - m_2^2)(m_1^2 - m_3^2)(m_1 + m_4)} \\ &- \frac{m_2 m_3 + m_1(m_2 + m_3)}{m_1(m_1^2 - m_2^2)(m_1^2 - m_3^2)(m_1 - m_4)} + \frac{m_2 m_3 + m_1(m_2 + m_3)}{m_3(m_1^2 - m_3^2)(m_2^2 - m_3^2)(m_3 + m_4)} \\ &+ \frac{1}{m_1 m_2 m_3 m_4}\Big)\,, \end{aligned} \tag{A.8}$$

$$
\begin{aligned}
N^L_{1234} = &\frac{1}{2(m_1^2-m_2^2)^2(m_1^2-m_3^2)^2(m_1^2-m_4^2)^2} \\
&\times\Big(6m_1^7m_3+(m_2-m_4)(m_2^2m_3^3m_4^2+3m_1^6m_3)-m_1m_2m_3^3m_4(m_2-m_4)^2+ \\
&+m_1^2m_3^2(m_2^3(2m_4+m_3)-m_4^3(2m_2+m_3)) \\
&-m_1^4(m_2-m_4)(2m_2^2(m_3+m_4)+m_2m_4(m_3+2m_4)+2m_3(m_3^2+m_4^2)) \\
&+m_1^3(-m_3^2m_4^3+m_2^2m_4^2(2m_3+m_4)+m_2m_3m_4(2m_3^2+m_4^2) \\
&+m_2^3(-m_3^2+m_3m_4+m_4^2)) \\
&-m_1^5(m_2^2(4m_3+m_4)+m_2(-m_3^2+2m_3m_4+m_4^2)+m_3(2m_3^2-m_3m_4+4m_4^2))\Big).
\end{aligned}
\tag{A.9}
$$

It is regular at coinciding arguments and has the symmetries

$$
N_{1234} = N_{\underline{1234}}, \quad N_{1234} = N_{234\underline{1}}, \quad N_{1234} = -N_{4321}. \tag{A.10}
$$

Similar to the case of N_{123} we can define $N^{(0)}_{1234}$, $N^{(12)}_{1234}$, $\ldots$, $N^{(34)}_{1234}$, $N^{(1234)}_{1234}$, such that $N^{(0)}_{1234}$, $f^{(pq)}_{1234} = N^{(pq)}_{1234}/m_pm_q$, and $f^{(1234)}_{1234} = N^{(1234)}_{1234}/m_1m_2m_4m_4$ are functions of m_1^2, $\ldots$, m_4^2, with

$$
N_{1234} = N^{(0)}_{1234} + \sum_{p<q} f^{(pq)}_{1234}m_pm_q + f^{(1234)}_{1234}m_1m_2m_3m_4. \tag{A.11}
$$

B. Γ_{gWZW}

The gauged Wess-Zumino-Witten action is given in [26] using the notation of differential forms, in addition to the earlier used notationl conventions in section 2. The following one-forms are introduced [26]:

$$
R = m^{-1}dm, \quad L = mdm^{-1}, \tag{B.1}
$$

$$
R_c = m^{-1}\hat{D}m = R + m^{-1}vm - v, \tag{B.2}
$$

$$
L_c = m\hat{D}m^{-1} = -\hat{D}mm^{-1} = L + mvm^{-1} - v = -mR_cm^{-1}. \tag{B.3}
$$

Here R_c and L_c are covariant under $U(n)\times U(n)$ gauge transformations. The extended gauged WZW action is given by

$$
\begin{aligned}
\Gamma_{\mathrm{gWZW}}[v,m] = &\frac{1}{48\pi^2}\int \mathrm{tr}\,\Big[-\frac{1}{5}R_c^5 + (R_c^3+L_c^3)F - 2(R_c+L_c)F^2 \\
&-R_cFm^{-1}Fm - L_cFmFm^{-1} - 4vF^2 + 2v^3F - \frac{2}{5}v^5\Big].
\end{aligned}
\tag{B.4}
$$

The integral is over a five-dimensional manifold with the physical four-dimensional space-time as boundary (the fields have been extended into a fifth dimension, v_5 can be taken equal to zero). Most of the integrations over the fifth dimension can be done, except for a term involving the WZW five-form R^5, and an equivalent expression [26] for a Γ_{gWZW} is given in (5.2).

The last three terms in (B.4) are not gauge invariant, they correspond to the $U(n)\times U(n)$ chiral anomaly. Their reduction to the Standard Model should be gauge invariant,

since the SM is anomaly free [27, 28, 29], which can be seen as follows. The potentially non-invariant terms constitute the Chern-Simons form, with exterior derivative

$$d\,\mathrm{tr}\left(-4\mathsf{v}\mathsf{F}^2 + 2\mathsf{v}^3\mathsf{F} - \frac{2}{5}\,\mathsf{v}^5\right) = -4\,\mathrm{tr}\;\mathsf{F}^3. \tag{B.5}$$

Introducing a six-dimensional manifold with the 5D manifold as boundary, and extending the gauge field into this 6D domain, we may write

$$\int \mathrm{tr}\left(-4\mathsf{v}\mathsf{F}^2 + 2\mathsf{v}^3\mathsf{F} - \frac{2}{5}\,\mathsf{v}^5\right) = -4\int \mathrm{tr}\;\mathsf{F}^3. \tag{B.6}$$

Writing the gauge field in terms of its generators, $F_{\mu\nu} = F^p_{\mu\nu}T_p$, we have

$$\mathrm{tr}\;\mathsf{F}^3 = \mathrm{str}(T_pT_qT_r)\,\mathsf{F}^p_{\kappa\lambda}\mathsf{F}^q_{\mu\nu}\mathsf{F}^r_{\rho\sigma}\,dx^\kappa\wedge dx^\lambda\cdots\wedge dx^\sigma, \tag{B.7}$$

in which $\mathrm{str}(T_pT_qT_r)$ is the symmetrized trace (since only the part of the trace that is symmetric under permutations of p, q and r contributes). For an anomaly-free representation of the gauge group $\mathrm{str}(T_pT_qT_r) = 0$, and for the reduction to the Standard Model Γ_{gWZW} is gauge invariant.

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